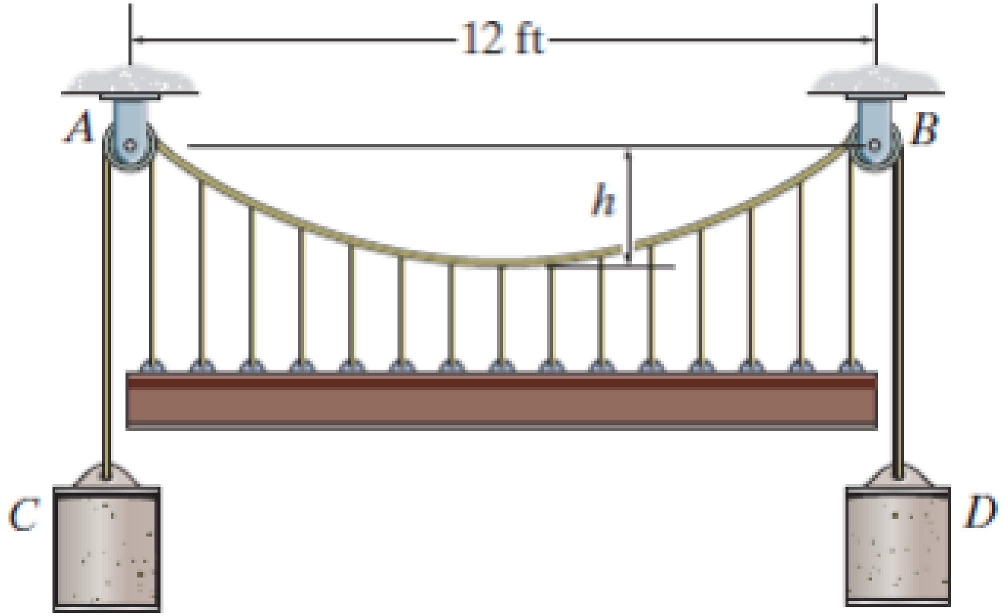
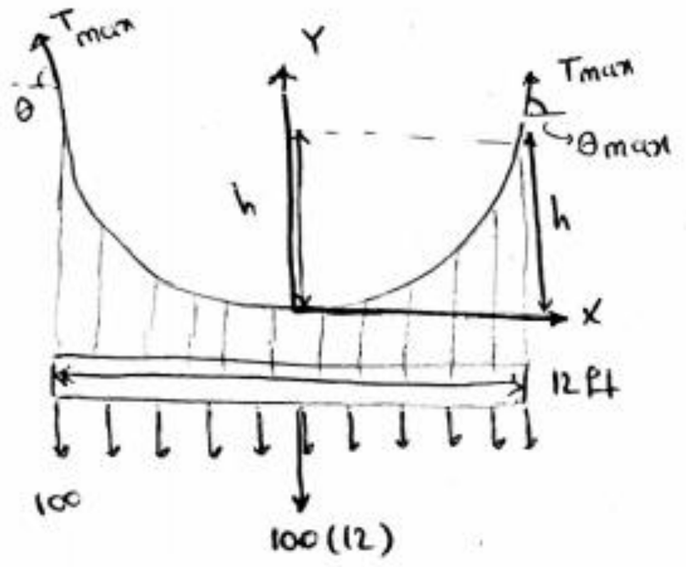


اگر وزن کیسه های C و D برابر 900 lb بوده و وزن واحد طول میله متصل به کابل برابر 100 lb/ft باشد، ماکزیمم شکم کابل (h) و طول کابل بین نقاط A و B را به دست آورید.

(راهنمایی: برای بدست آوردن شکم و طول کابل لازم است تابع شکل کابل را بدست آورید.)



$T_{max} = 900 \text{ lb}$

$\Sigma F_y = 0 \qquad 2(900 \sin\theta_{max}) - 100(12) = 0 \qquad \theta_{max} = 41.81$

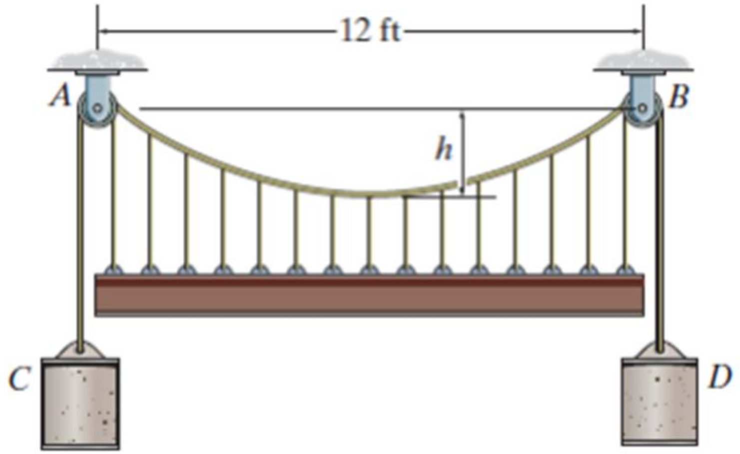
$T_0 = T_{max} \cos\theta_{max} = 900 \cos(41.81) = 670.82 \text{ lb}$

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{100}{670.82} = 0.1491$$

$$\frac{dy}{dx} = 0.1491x + C_1 \quad \xrightarrow[dy/dx = 0]{x=0} \quad C_1 = 0$$

$$y = 0.1491 \, x^2/2 + C_2 \quad \xrightarrow[y=0]{x=0} \quad C_2 = 0$$

$$\boxed{y = 0.07454x^2}$$

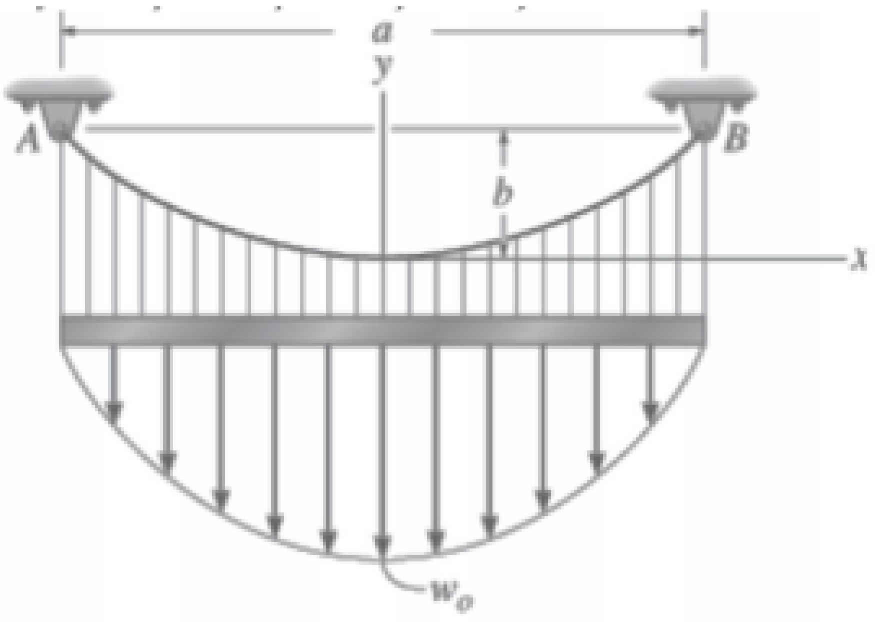


$$y = h\Big|_{@x=6} \qquad h = 0.07454(6^2) = 2.68 \text{ ft}$$

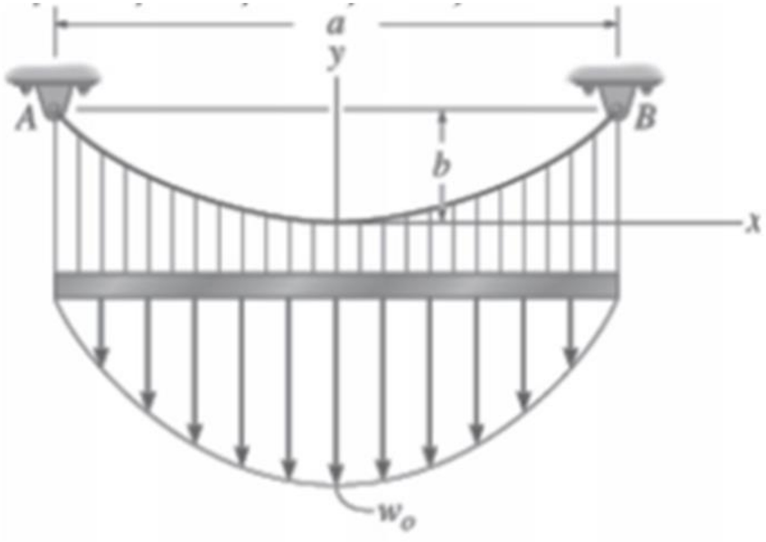
$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{1 + 0.02222x^2}\, dx$$

$$\text{طول } L = \int ds = 2\int_0^6 \sqrt{1 + 0.02222x^2}\, dx = 0.2981 \int_0^6 \sqrt{45 + x^2}\, dx$$

$$L = 0.2981 \left\{ \frac{1}{2}\left[x\sqrt{45 + x^2} + 45 \ln\left(x + \sqrt{45 + x^2}\right)\right]\right\}\Big|_0^6 = 13.4 \text{ ft}$$



کابل در شکل مقابل تحت بار سهموی $w = w_0(1-(2x/a)^2)$ قرار دارد. بیشترین نیروی موجود در کابل و همچنین معادله تعیین‌کننده شکل کابل AB را به‌دست آورید.



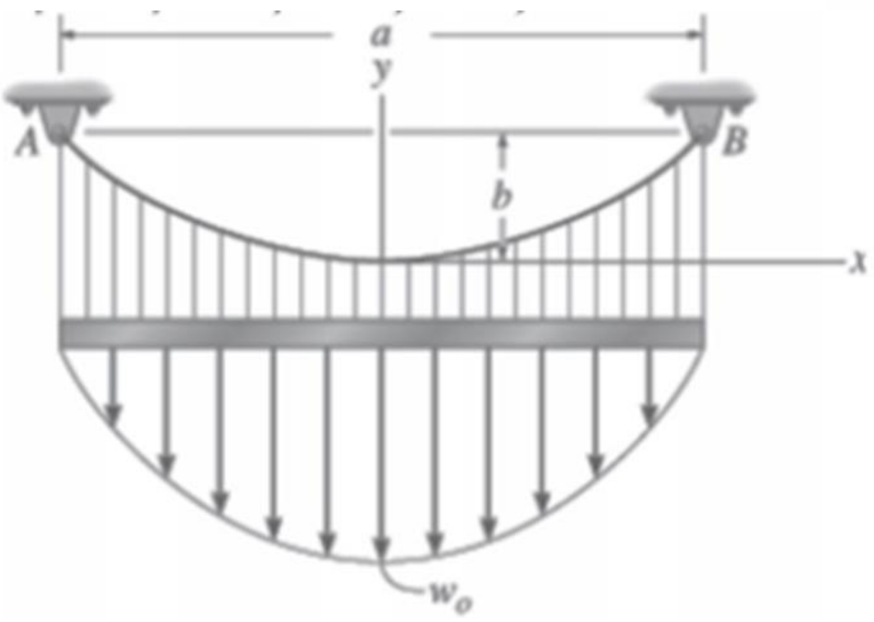
$w = w_o\left(1-\left(\frac{2x}{a}\right)^2\right)$ $\qquad$ $d^2y/dx^2 = \frac{w(x)}{T_o} = \frac{w_o}{T_o}\left(1-\left(\frac{2x}{a}\right)^2\right)$

$y' = \frac{1}{T_o}\left(w_o\left(x - \frac{4x^3}{3a^2}\right)\right) + C_1$ $\qquad$ $x=0 \quad y'=0 \longrightarrow C_1 = 0$

$y = \frac{1}{T_o}\left(w_o x^2/2 - w_o \frac{x^4}{3a^2}\right) + C_2$ $\qquad$ $x=0 \quad y=0 \longrightarrow C_2 = 0$

$y = \frac{1}{T_o}\left(w_o x^2/2 - w_o x^4/3a^2\right)$ $\qquad$ $y' = \frac{1}{T_o}\left(w_o x - \frac{4x^3}{3a^2} w_o\right)$

$x = a/2 \quad y = b$ $\qquad$ $b = \frac{1}{T_o}\left(\frac{w_o}{2}\left(\frac{a}{2}\right)^2 - w_o \frac{1}{3a^2}\left(\frac{a}{2}\right)^4\right) \longrightarrow b = \frac{5}{48}\left(\frac{w_o a^2}{T_o}\right)$ $\quad$ $T_o = \frac{5w_o a^2}{48b}$

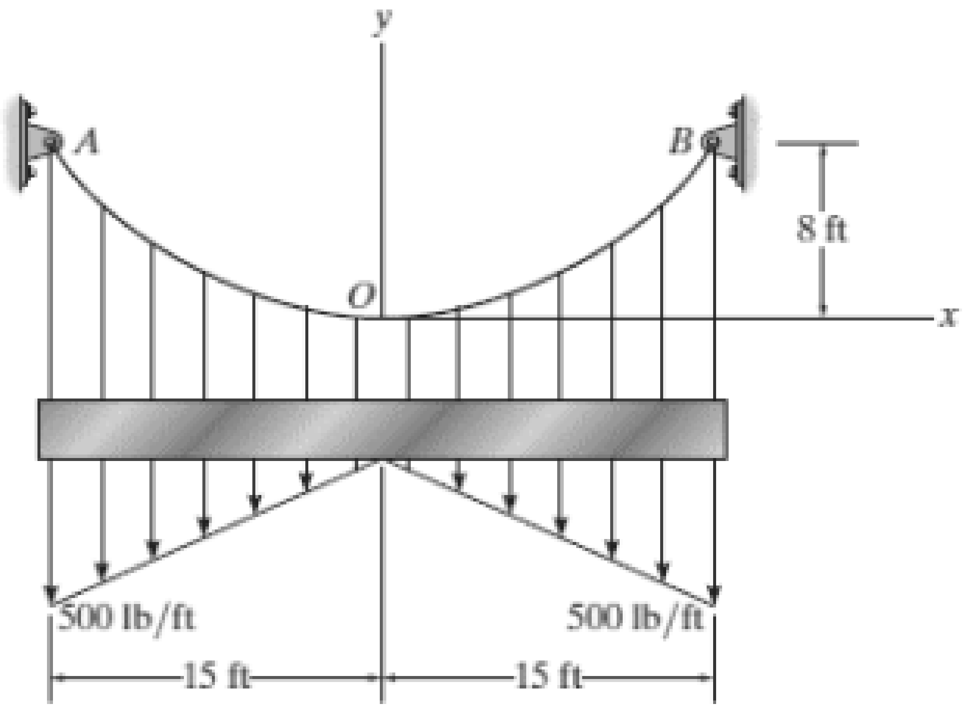


$$y(x) = \frac{48b}{5w_o a^2}\left(w_o\left(\frac{x^2}{2} - \frac{x^4}{3a^2}\right)\right) = \frac{24b}{5a^2}x^2 - \frac{16b}{5a^4}x^4$$

$$tg(\theta max) = \left.\frac{dy}{dx}\right|_{x=\frac{a}{2}} = \frac{1}{T_o}\left[w_o\frac{a}{2} - \frac{4w_o}{3a^2}\left(\frac{a}{2}\right)^3\right] = \frac{w_o a}{3T_o}$$

$$\theta max = tg^{-1}\left(\frac{w_o a}{3T_o}\right) = tg^{-1}\left(\frac{16b}{5a}\right)$$

$$T_{max} = \frac{T_o}{\cos\theta_{max}} = \frac{5w_o a^2}{48b\cos\left(tg^{-1}\left(\frac{16b}{5a}\right)\right)}$$



۲- در کابل روبرو اگر شیب کابل در نقطه O صفر باشد، رابطه شکل کابل y=f(x) و کشش ماکزیمم در کابل را بدست آورید.

$w(x) = \frac{500}{15} x \qquad d^2y/dx^2 = \frac{w(x)}{T_0} \qquad d^2y/dx^2 = \frac{500}{15} x \frac{1}{T_0} \qquad \frac{dy}{dx} = \frac{1}{T_0} \frac{500}{15} \frac{x^2}{2} + C_1$

$x = 0 \quad dy/dx = 0 \longrightarrow C_1 = 0$

$y = \frac{1}{T_0} \frac{500}{15} x^3/6 + C_2 \qquad x = 0 \quad y = 0 \longrightarrow C_2 = 0 \qquad y = \frac{1}{T_0}\left(\frac{50}{9} x^3\right)$

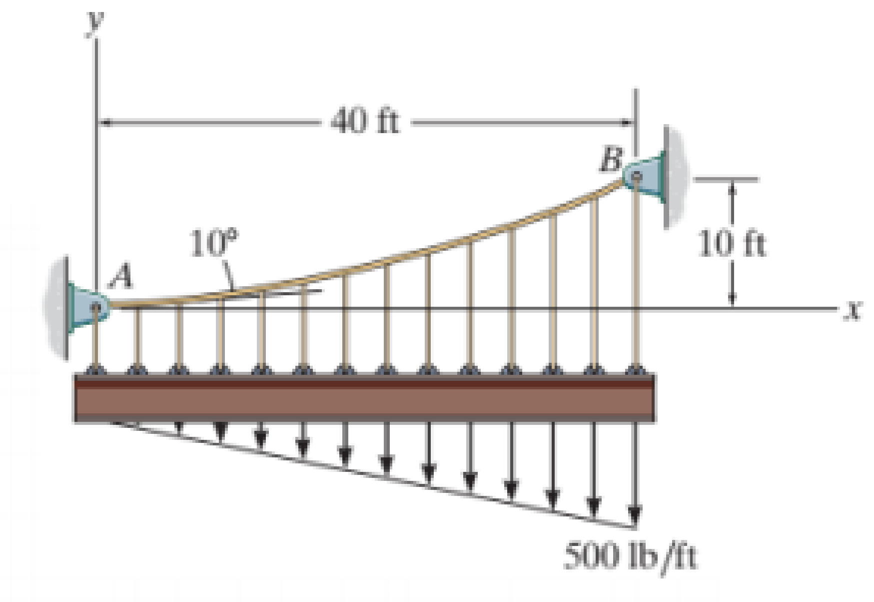
$x = 15 \quad y = 8 \qquad 8 = \frac{1}{T_0}\left(\frac{50}{9}(15)^3\right) \longrightarrow \boxed{T_0 = 2344 \text{ lb}}$

$$\left.dy/dx\right|_{x=15\text{ ft}} = tg(\theta_{max}) = \left.\frac{50}{3(2344)} x^2\right|_{x=15} = 1.6$$

$$\theta_{max} = tg^{-1}(1.6) = 57.99^\circ$$

$$T_{max} = \frac{T_0}{\cos\theta_{max}} = \frac{2344}{\cos(57.99)} = 4422 \text{ lb}$$

۲- رابطه شکل کابل و ماکزیمم کشش ایجاد شده در کابل مقابل را بدست آورید.



$$w(x) = \frac{500}{40} x = 12.5x \qquad d^2y/dx^2 = \frac{w(x)}{T_0} = \frac{12.5}{T_0} x \qquad dy/dx = 6.25 \frac{x^2}{T_0} + C_1$$

$$x = 0 \,,\; dy/dx = tg(10^\circ) \longrightarrow C_1 = tg(10^\circ)$$

$$y = \frac{2.0833}{T_0} x^3 + tg(10^\circ)\, x + C_2 \qquad x = 0 \quad y = 0 \longrightarrow C_2 = 0$$

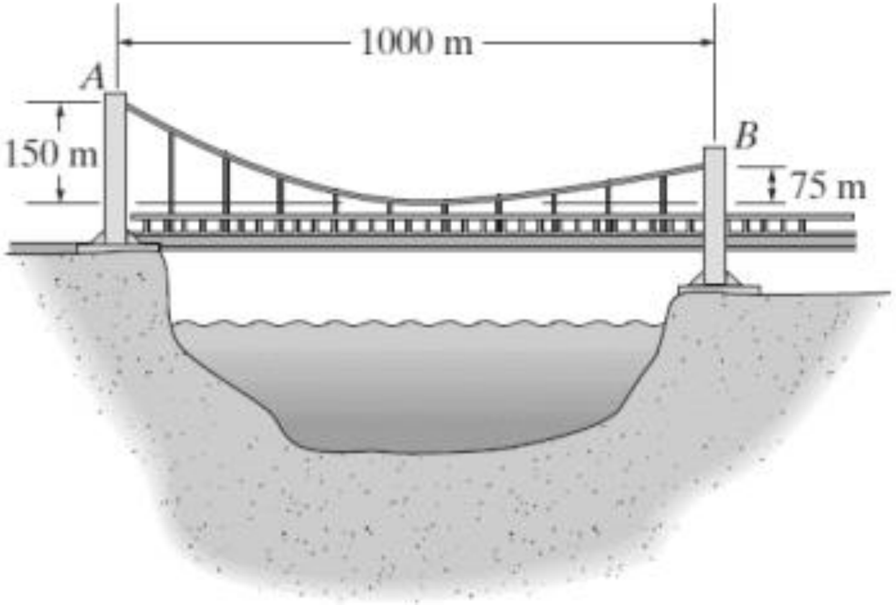
$$y = \frac{2.0833}{T_0} x^3 + tg(10^\circ)\, x \qquad x = 40 \text{ ft} \quad y = 10 \text{ ft}$$

$$10 = \frac{2.0833}{T_0} (40)^3 + 10\, tg(10^\circ) \longrightarrow T_0 = 45.245 \times 10^3 \text{ ft}$$

$$y = 46 \times 10^{-6} x^3 + 0.176\, x \qquad \theta = dy/dx = 0.1381 \times 10^{-3} x^2 + 0.176$$

$$\theta_{max} = tg^{-1}\left(\left.\frac{dy}{dx}\right|_{x=40} \right) = tg^{-1}\left(0.1381(10^{-3})(40^2) + 0.176 \right) = 21.67^\circ$$

$$T_{max} = \frac{T_o}{\cos\theta_{max}} = \frac{45.245 \times 10^3}{\cos(21.67)} = 48.69 \times 10^3 \text{ lb}$$



۴ - پل دارای وزن بر واحد طول $80\,\frac{kN}{m}$ می‌باشد. وزن پل توسط کابل که به دو طرف وصل شده است، تحمل می‌گردد.کشش کابل را در در نقطه A و B بدست آورید.

$W(x) = \frac{80}{2} = 40$ $\quad d^2y/dx^2 = \frac{w(x)}{T_0} = \frac{40 \times 10^3}{T_0}$

$dy/dx = \frac{40 \times 10^3}{T_0} x + C_1$ $\quad x = 0 \quad dy/dx = 0 \longrightarrow C_1 = 0$

$y = \frac{20 \times 10^3 x^2}{T_0} + C_2$ $\quad x = 0 \quad y = 0 \longrightarrow C_2 = 0$ $\quad \boxed{y = \frac{20 \times 10^3}{T_0} x^2}$

$x = x_0 \quad y = 75 \longrightarrow 75 = \frac{20 \times 10^3}{T_0} x_0^2$

$x = -(1000 - x_0) \quad y = 150 \longrightarrow 150 = \frac{20 \times 10^3}{T_0} (-(1000 - x_0))^2$ $\quad x_0 = 414.21$

$T_0 = 45.75 \times 10^6$

$$dy/dx = \frac{40 \times 10^{3}}{45.75 \times 10^{6}} x$$

$$\theta_B = tg^{-1}\left(dy/dx \Big|_{x_B} \right) = tg^{-1}\left(0.8743 \times 10^{-3} (414.21)\right) = 19.91$$

$$\theta_A = tg^{-1}\left(dy/dx \Big|_{x_A} \right) = tg^{-1}\left(0.8743 \times 10^{-3} (-(1000-414.21))\right) = 27.12$$

$$T_B = \frac{T_o}{\cos\theta_B} = \frac{45.75 \times 10^{6}}{\cos 19.91} = 48.66 \times 10^{6} \text{ N}$$

$$T_A = \frac{T_o}{\cos\theta_A} = \frac{45.75 \times 10^{6}}{\cos 27.12} = 51.4 \times 10^{6}$$